

# Musical Abacus: A Comprehensive Tool for Music Education

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## Abstract

Together with its creative side, music also has a purely technical, even mathematical, side that includes intervals, scales, keys, chords, etc. that sets it apart from other arts. With the objective of unifying all these technical contents, introducing them in a logical and orderly way, the so called Musical Abacus has been designed, a comprehensive tool intended for music education. It contains the information on the twelve possible keys and scales, both major and minor, as well as on harmonics, intervals, and chords. All these concepts are basic in any musical style, such as: classical music, modern music, Jazz, Latin music, etc. Guidelines to logically connect those concepts are also given. Moreover, in order to use the Musical Abacus, users need not know how to read music. It has been presented in several Conservatories and musical societies, and it has been greatly appreciated. In fact, it is currently used in some music schools.

## Keywords

Music Theory, Harmonic, Interval, Key, Scale, Chord

## Introduction

Together with its creative side, music also has a purely technical, even mathematical, side that includes intervals, scales, keys, chords, etc. that sets it apart from other arts. The study of this technical part is quite complex and usually takes several years. This results in that some questions are forgotten at the time as others are learnt. Furthermore, most of this part is focused in those keys having few accidentals in their key signature, thus the number of musicians fully dominating the twelve keys is scarce.

With the objective of unifying all those technical contents, introducing them in a logical and orderly way, and giving all the keys the same relevance, the so called *Musical Abacus* has been designed, a comprehensive tool that can be considered as a real “musical calculator”. It consists of two rotating discs, one being cardboard and the other plastic. Both discs are 12-sided polygons (or dodecagons). This is due to the fact that there are 12 different musical notes and, therefore, 12 major keys, 12 minor keys, 12 major chords, 12 minor chords, etc.

In each of the 12 positions that can be selected we obtain:

- 1) A major key and its relative minor.
- 2) The corresponding major scale and its relative minor.
- 3) The intervals from the tonic of the major scale.
- 4) The 3 main chords in both keys with the notes forming them.
- 5) The harmonics produced by the tonic of the major key.
- 6) The corresponding key signature for both keys.

All these concepts are basic in any musical style, such as: classical music, modern music, Jazz, Latin music, etc. Consequently, they are explained in any general treatise on music theory or harmony, such as, for example, Danhauser (1975) or Schenker (1954). The Musical Abacus has been presented in

several Conservatories and musical societies, and it has been greatly appreciated. In fact, it is currently used in some music schools. Moreover, in order to use it, users need not know how to read music. As an example, Fig. 1 shows the Musical Abacus in the C major / A minor position.

Next, instructions on how to interpret all the information given by this tool are fully explained. They can also be used as guidelines to logically connect the different musical concepts.

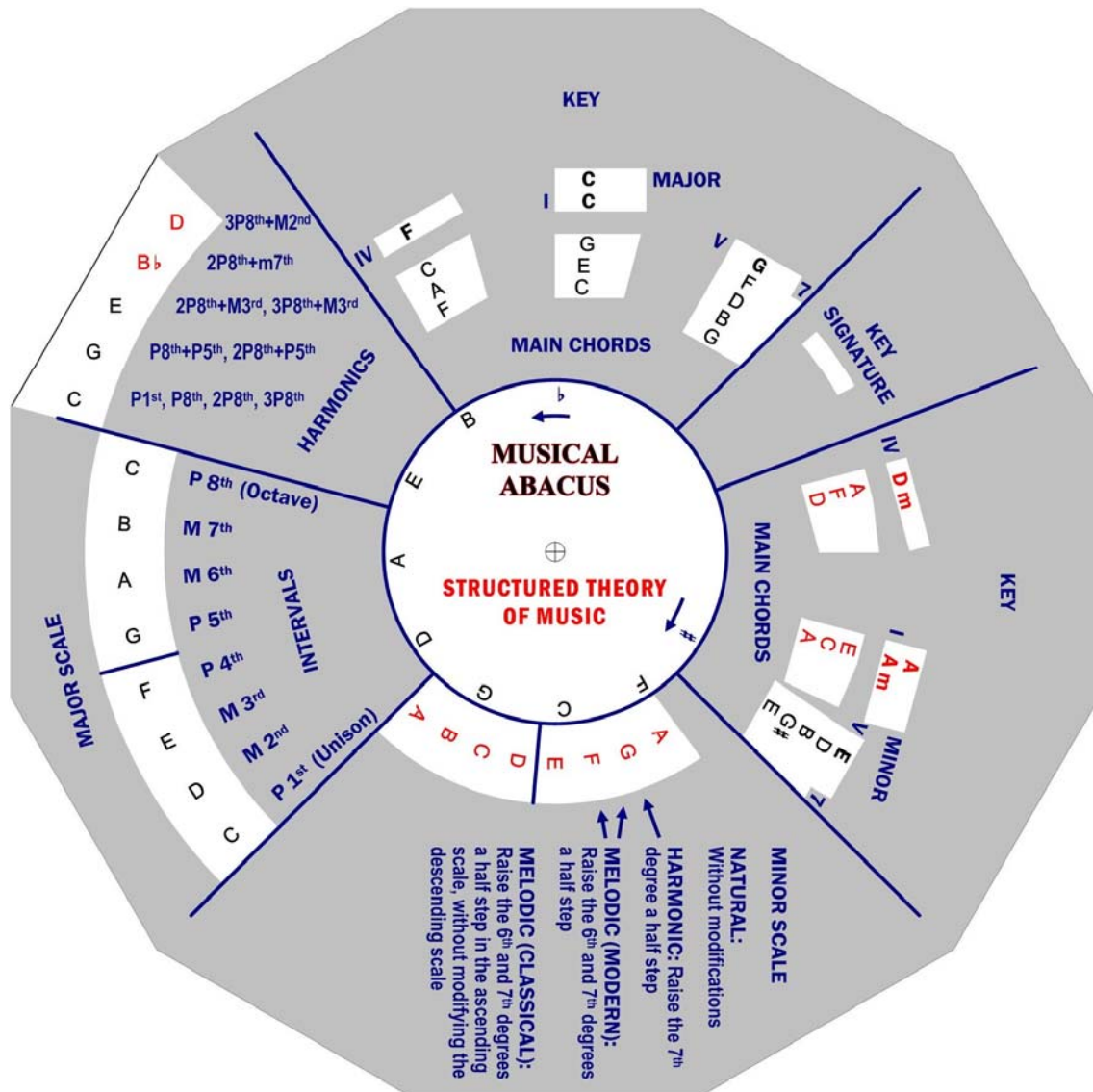


Figure 1. The Musical Abacus in the C major / A minor position.

## Harmonics

The theory of harmonics is of capital importance for organising the musical concepts. Unfortunately, it is usually not known in detail by most musicians. In short, it states that, when any note is played on an instrument, other higher notes are automatically produced, which sound together with it. These notes are called *harmonics*, *overtones* or *partials* and make up a *harmonic series*. For instance, if note C is played, the first 10 harmonics produced are: 1-C, 2-C, 3-G, 4-C, 5-E, 6-G, 7-Bb, 8-C, 9-D, 10-E (in this study, we neglect the small differences in pitch existing between the harmonics and the real tuning of the instrument, which may be, for example, the equal temperament). Harmonic 1-C is the actual played note and is called *fundamental*. The Musical Abacus gives us the first 10 harmonics of any

note. For example, Fig. 1, upper left area or “Harmonics” area, shows the first 10 harmonics of note C (the harmonics corresponding to notes with the same name are grouped together).

It can be proved that odd harmonics (in this case, 1-C, 3-G, 5-E, 7-Bb, 9-D) always correspond to different notes, while even harmonics (in this case, 2-C, 4-C, 6-G, 8-C, 10-E) correspond to notes of previous odd harmonics. The relative intensities (strengths) of the harmonics determine the *timbre* of the instrument, and generally the higher the harmonic, the lesser the intensity.

The habit of constantly hearing the harmonic series has resulted in:

- 1) Notes corresponding to harmonics 1 and 2 (a perfect 8<sup>th</sup> apart) are perceived so affine or consonant that they are given the same name (in this example, C). This is also applicable to any pair of harmonics showing the ratio 2:1, such as 4:2, 8:4, etc. It means that notes corresponding to harmonics 4, 8, etc., are given the same name as harmonics 1 and 2 (that is, C).
- 2) Notes corresponding to harmonics 1 and 3 (a perfect 5<sup>th</sup> apart) are considered very affine or consonant between them. This is also applicable to harmonics 1 and 6, since harmonics 6 and 3 show the ratio 2:1, and therefore they are given the same name (in this example, G).
- 3) Notes corresponding to harmonics 1 and 5 (a major 3<sup>rd</sup> apart) are also considered very affine or consonant between them (although to a lesser extent than harmonics 1 and 3). This is also applicable to harmonics 1 and 10, since harmonics 10 and 5 show the ratio 2:1, and therefore they are given the same name (in this example, E).
- 4) Notes corresponding to harmonics 3 and 5 (a major 6<sup>th</sup> apart) are also considered affine or consonant between them. This is also applicable to harmonics 6 and 10, for the reasons just given.

Consequently, apart from the unison and the octave, the *consonant intervals* will be the perfect 5<sup>th</sup>, the major 3<sup>rd</sup> and the major 6<sup>th</sup>. The *inversion of the intervals*, which is achieved by changing one of the notes an octave, gives the perfect 4<sup>th</sup>, minor 6<sup>th</sup> and minor 3<sup>rd</sup> intervals, respectively, being all of them consonant as well. The rest of the intervals are considered *dissonant*. Therefore, in the harmonic series considered above, the intervals between any two of harmonics 1-C, 3-G, 5-E are consonant, while the intervals between 1-C and harmonics 7-Bb and 9-D are dissonant. This is the reason why harmonics 7 and 9 are represented in different colour in Fig. 1.

## Major Chords, Keys and Scales

A *major chord* is formed by the first 3 notes, different among them, of a harmonic series. Thus, the C major chord is formed by harmonics 1-C, 3-G and 5-E of C. It is simply represented by C. Since the 3 intervals formed among the notes of a major chord are consonant, the chord is consonant, too.

A *major key* consists of the notes of 3 major chords, one “central” and the other two a perfect 5<sup>th</sup> above and below it. We can obtain the last 2 chords with the Musical Abacus by simply rotating the plastic disc (with respect to the cardboard disc) one step to the right or one step to the left, respectively. For example, if the central chord is C = {C, E, G}, the other 2 chords will be G = {G, B, D} and F = {F, A, C}, respectively. Then, the C major key consists of the notes of these 3 chords, which are 7 (apparently, they are 9, but 2 of them, C and G, are repeated). Fig. 1, upper area or “Major key” area, shows the name of the key (C mayor), the names of the 3 chords (F, C, G) and, just below, the notes forming them. In the case of chord G an extra note is added, the one corresponding to the next harmonic in the series of G, that is, 7-F, which also belongs to C mayor key. This is a common practice and the resulting chord is called G *dominant seventh*, because note F forms an interval of (minor) 7<sup>th</sup> with the *root* of the chord, G. As this interval is dissonant, the chord is dissonant, too. It is represented by G7. If we try to do the same process with chords F and C, we would obtain the extra notes Eb and Bb, respectively, which do not belong to C mayor key. Therefore, it is not so common to

add them to the chords and they are not included in Fig. 1. Thus, the *main chords* in C major key are C, F and G7, as shown in Fig. 1.

If we sort the notes of a major key by their pitch we obtain the corresponding *major scale*. In our example, the C major scale will be formed by notes C, D, E, F, G, A, B. It can be seen in Fig. 1, left area or “Major scale” area, where the *Tonic* of the scale, C, is repeated at the end, as it is usually done in practice. If we assign Roman numerals to these notes, starting from I, they will define the *degrees* of the scale. The root notes of the main chords then correspond to degrees I, IV and V, as it is indicated just before the chord names.

## Intervals, Tetrachords

An *interval* is the distance in pitch between 2 notes, and they are named by a number and a quality. The quality of an interval is based on the structure of the major scale, so that all intervals between the tonic and any other note in that scale are perfect or major. Taking these kinds of intervals as a reference, the other kinds of intervals (minor, augmented and diminished) are easily defined. The Musical Abacus gives the information on intervals along with the major scale (Fig. 1, Major scale area), showing the intervals from the tonic to the rest of the notes in the scale.

The notes of a major scale can be grouped into 2 *tetrachords* with the same intervallic structure, which is WWH (W stands for Whole step and H stands for Half step), the distance between the 2 tetrachords being W. This fact allows to connect any major scale with other 2. Thus, for example, in Fig. 1, Major scale area, the C major scale is divided by a line into tetrachords {C, D, E, F} and {G, A, B, C}. When rotating the plastic disc one step to the right, the 2<sup>nd</sup> tetrachord of C major becomes the 1<sup>st</sup> tetrachord of G major. And, when rotating it one step to the left, the 1<sup>st</sup> tetrachord of C major becomes the 2<sup>nd</sup> tetrachord of F major. This way, the major scales (and so, the keys) are sorted by perfect 5<sup>th</sup> or, in other words, according to the *cycle of fifths*.

## Minor Chords, Keys and Scales

For any given note, a major chord can be obtained by superimposing a mayor 3<sup>rd</sup> on it, and then a minor 3<sup>rd</sup> on the note thus obtained. But, if we first superimpose a minor 3<sup>rd</sup> and then a mayor 3<sup>rd</sup>, the result is another consonant chord: the *minor chord*. For example, the A minor chord is formed by notes A, C, E. It is represented by Am. Since the 3 intervals formed among the notes of a minor chord are consonant, the chord is consonant, too. As has been seen, a minor chord is obtained in an artificial way, that is, not following a harmonic series.

Similarly to the major key, a *minor key* consists of the notes of 3 minor chords, one central and the other two a perfect 5<sup>th</sup> above and below it. Thus, the A minor key will consist of the notes of chords Am = {A, C, E}, Em = {E, G, B} and Dm = {D, F, A}, which are 7 notes (apparently, they are 9, but 2 of them, A and E, are repeated). These notes, sorted by their pitch, are: A, B, C, D, E, F, G and make up the corresponding *natural minor scale*. Note that these notes are the same as in the C major key, so it is said that C major and A minor are *relative keys* to each other. In the Musical Abacus, every two relative keys are shown together. For example, Fig. 1 shows simultaneously C major and A minor keys. When we are interested in a minor key, it is preferable to turn the whole Musical Abacus ninety degrees counterclockwise, so that the “Minor key” area (right area in Fig. 1) moves to the upper area and the “Minor scale” area (bottom area in Fig. 1) moves to the right area.

The natural minor scale has, however, an inconvenience: the distance between its VII and VIII degrees (notes G and A in the last example) is a whole step instead of a half step, as occurs in a major scale. This results in that the VII degree does not show “attraction to the Tonic”, which means that when playing this scale and passing from the VII to the VIII degree, it does not produce the psychological sensation of having reached the end of the scale. To avoid this inconvenience, it is common to raise a half step the VII degree (giving G#), which results in the *harmonic minor scale*. This is the most used

version of the minor scale, in which the minor chord  $Em = \{E, G, B\}$  is turned into the major chord  $E = \{E, G\#, B\}$ . And, as in the major key, it is common to add to this chord the harmonic 7 of its root (in this case, E), that is, note D, which also belongs to A minor key, giving the E7 chord.

The harmonic minor scale has, nonetheless, a further inconvenience: the interval between its VI and VII degrees is an augmented 2<sup>nd</sup>, which produces a strange and unnatural effect, for it is a too big interval to be between two consecutive degrees. So, sometimes the VI degree is also raised a half step (giving F#), thus solving this problem and resulting in the *melodic minor scale*. Now, as raising the VI and VII degrees is only needed in the ascending scale, but not in the descending, we find two different versions for this scale: In a *classical* context, it is usually understood that the melodic minor scale is that having its VI and VII degrees raised a half step in the ascending scale, leaving the descending unmodified (that is, equal to the natural minor scale). On the other hand, in a *modern* context, it is usually understood that the melodic minor scale is that having its VI and VII degrees raised a half step both in the ascending and descending scales.

Fig. 1, Minor key area, shows the name of the key (A minor), the names of the 3 main chords in the corresponding harmonic minor scale (Dm, Am, E7) and, just below, the notes forming them. Additionally, the Minor scale area shows the natural minor scale, with the tonic, A, repeated at the end, as it is usually done in practice. Next to it, directions for obtaining both the harmonic and the melodic minor scales are given (in the last case, including both the classical and the modern versions). As in the major scale, if we assign Roman numerals to these notes, starting from I, they will define the degrees of the scale. The root notes of the main chords then correspond to degrees I, IV and V, as it is indicated just before the chord names.

Similarly to the major scale, the A natural minor scale is divided by a line into 2 tetrachords. In this case, however, their intervallic structures are different, so minor scales cannot be connected in the same way as major scales.

### **The Rest of the Keys, Key Signatures, Order of Sharps and Flats**

The rest of mayor and minor keys and scales are obtained by rotating the 2 discs of the Musical Abacus between them. As explained above, they are sorted according to the cycle of fifths. For example, Fig. 2 shows the Musical Abacus in the D major / B minor position. On the upper right area or “Key signature” area, the number and kind of accidentals for these keys are shown (2#). And, in the central area, there is an arrow showing the notes affected by these accidentals, which are F (the first note in the series) and C (in general, the rest of notes up to the arrow). This means that these notes change to F# and C#.

Logically, in Fig. 1 the Key signature area is blank, because C major and A minor keys have no accidental. In this case, the arrows in the central area indicate the *order of sharps and flats*.

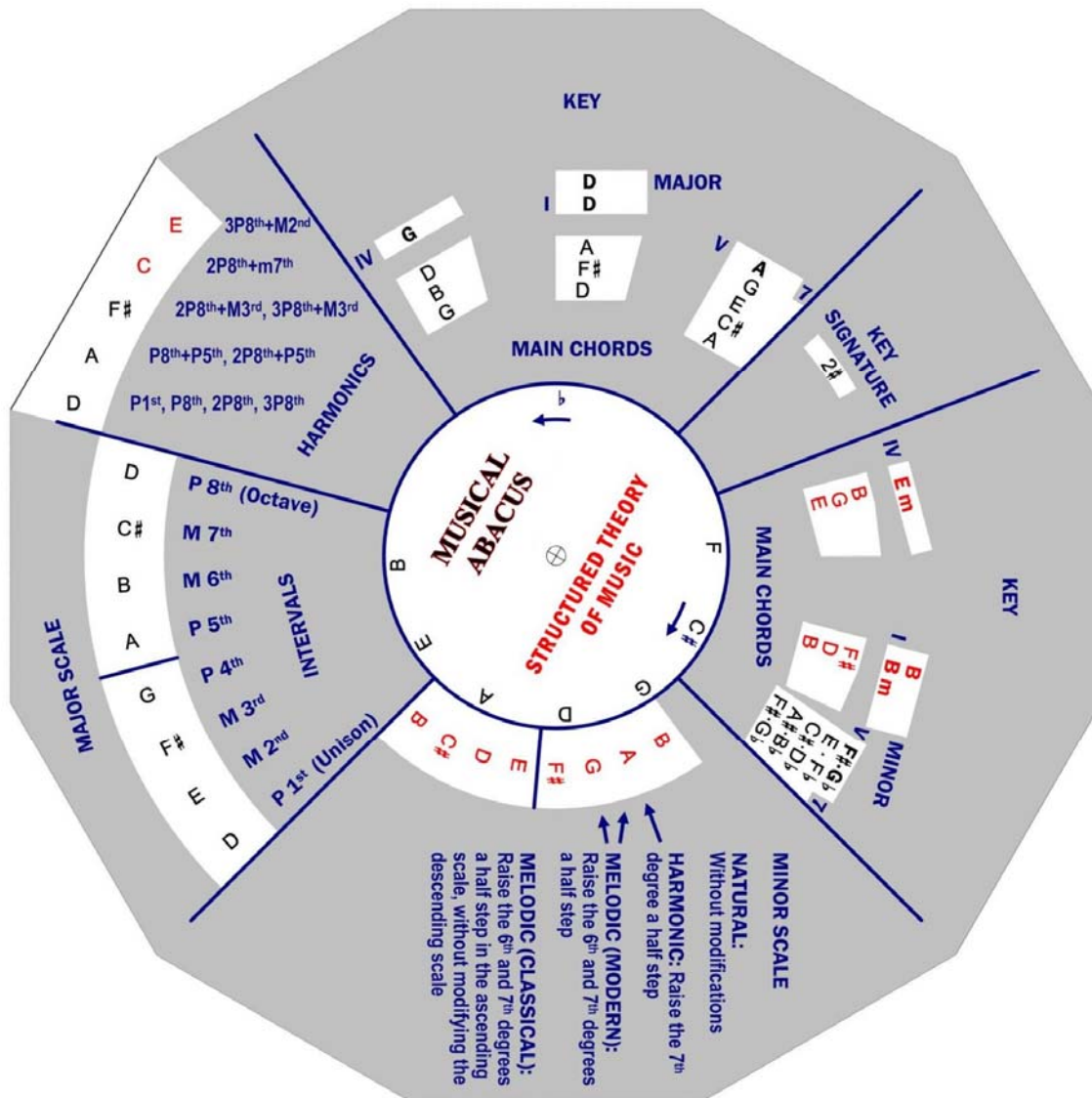


Figure 2. The Musical Abacus in the D major / B minor position.

## Conclusions

The Musical Abacus integrates the main concepts in Music Theory, relating them in a logical and ordered way. It contains the information on harmonics, intervals, keys, scales and chords, both major and minor. All these concepts are basic in any musical style, such as: classical music, modern music, Jazz, Latin music, etc. Moreover, in order to use it, users need not know how to read music. The Musical Abacus has been presented in several Conservatories and musical societies, and it has been greatly appreciated. In fact, it is currently used in some music schools.

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